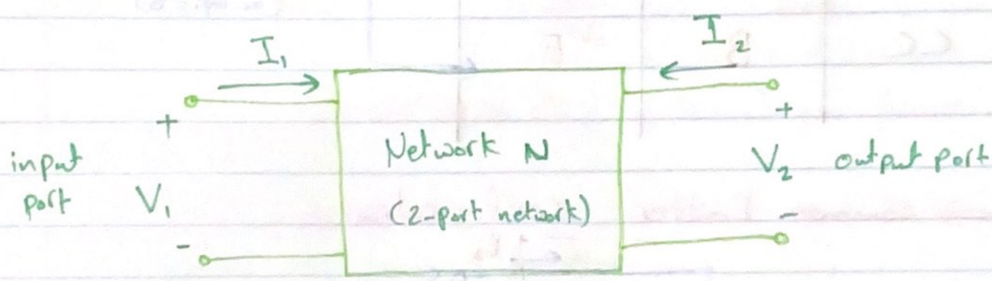
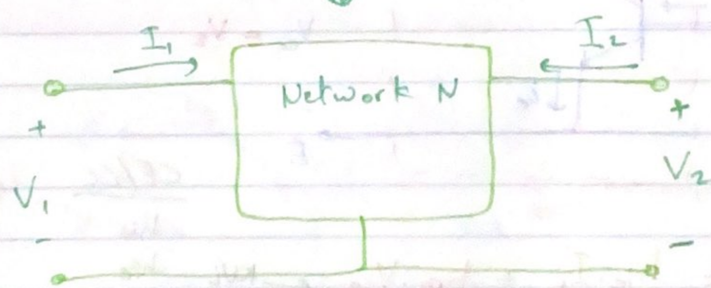
T8: BJT AC Models and Analysis

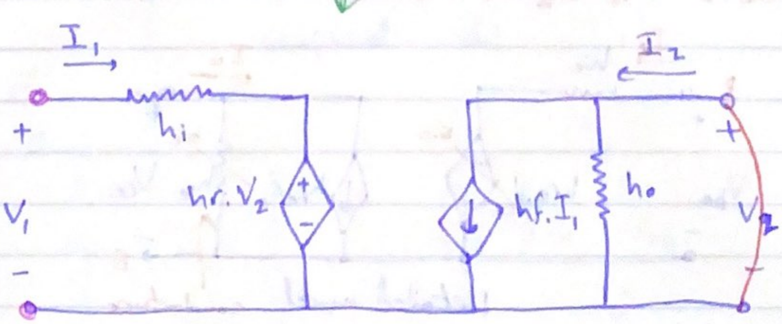
* we will deal with small signal amplifier (not power amplifier)

- There are two models used in small signal AC analysis of a transistor:
  1) r_e model X
  2) Hybrid equivalent model ** (h-parameter)

⇒ Two-port networks







h-parameter equations:

KVL → $V_1 = h_i I_1 + h_r V_2$

KCL → $I_2 = h_f I_1 + h_o V_2$

$h_i = \left. \frac{V_1}{I_1} \right|_{V_2=0}$ $\qquad h_r = \left. \frac{V_1}{V_2} \right|_{I_1=0}$

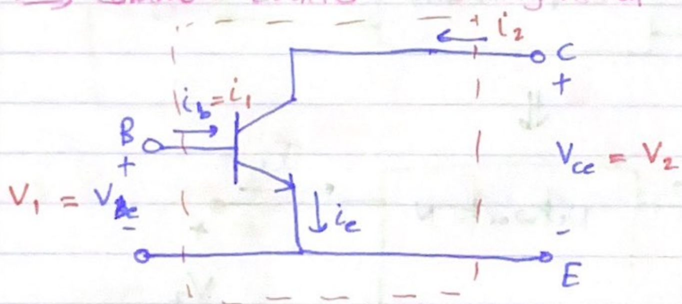
$h_f = \left. \frac{I_2}{I_1} \right|_{V_2=0}$ $\qquad h_o = \left. \frac{I_2}{V_2} \right|_{I_1=0}$

⟹ BJT configurations

	ac input	dc output
CE	B	C
CB	E	C
CC	B	E

same eq. circuit (ac inp. = B)

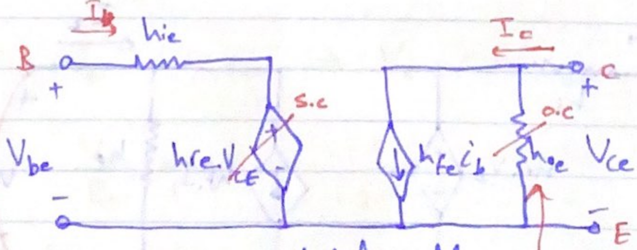
⟹ Common Emitter configuration



CE/CC	CB
h_{ie}	h_{ib}
h_{fe}	h_{fb}
h_{re}	h_{rb}
h_{oe}	h_{ob}

$V_{be} = h_{ie} \cdot I_b + h_{re} \cdot V_{ce}$ ← KVL

$I_c = h_{fe} \cdot I_b + h_{oe} \cdot V_{ce}$



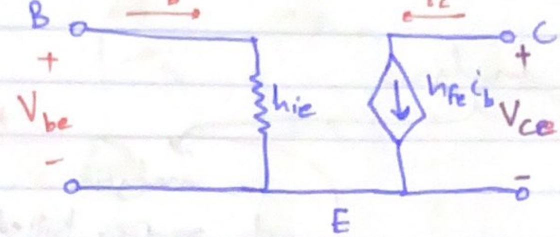
Detailed model

conductance

$\frac{1}{\Omega}$ = simens

$h_{re} \approx 0.0002$ X

$h_{oe} \approx 2 \times 10^{-6}$ siemens X simplified model



** will be used

$h_{fe} = \frac{i_c}{i_b} = \beta$

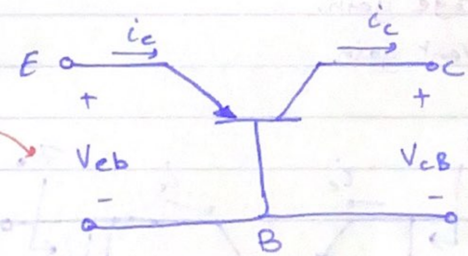
$$h_{ie} = \frac{V_T}{I_{BQ}} \Omega = \frac{V_T}{\frac{I_{CQ}}{h_{fe}}} = \frac{h_{fe} V_T}{I_{CQ}}$$

$V_T = 25.69\ mV$ @ 25 C°

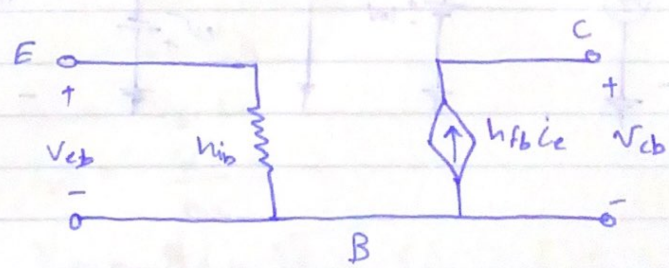
⇒ common collector

· Same as common emitter

⇒ common-Base configuration



simplified model



$h_{ib} = \frac{V_T}{I_{EQ}}$ $\quad h_{fb} = \alpha = \frac{i_c}{i_e}$ $\quad h_{ie} > h_{ib}$

⇒ BJT Amplifier Analysis

analysis

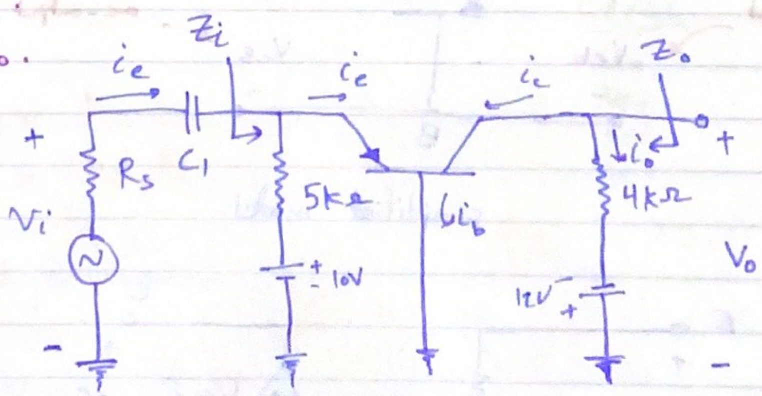
dc analysis
- ac source killed
- caps ~~circuit~~ open

ac analysis **
- dc sources killed
  - $V \to 0 \to$ short
  - $I \to 0 \to$ open
- $X_c = \frac{1}{2\pi f_c}$; $f_c \uparrow\uparrow$
  - $X_c \approx 0 \to$ caps short

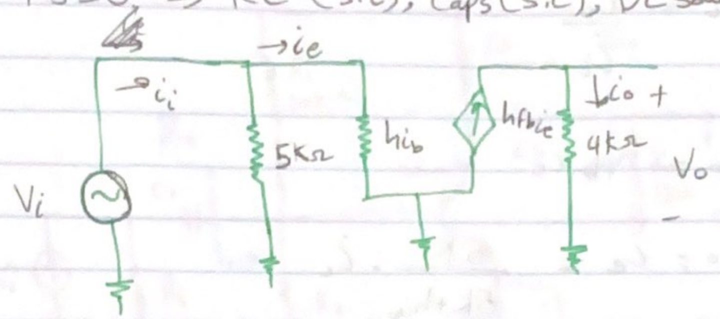
. small signal voltage gain $= A_v = \frac{V_o}{V_i}$

. small signal current gain $= A_i = \frac{i_o}{i_i}$

ex find all of the following quantities with and without Rs:
1) small signal voltage gain.
2) small signal current gain.
3) Z_i.
4) Z_o.



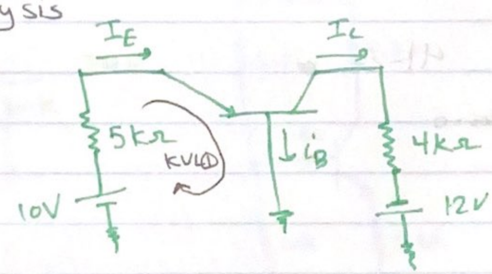
a) with $R_S = 0$, ⇒ RC (s.c), Caps (s.c), DC sources (killed)



. $h_{fb} = \alpha \cong 1$

$h_{ib} = \frac{V_T}{I_{EQ}}$; I_{EQ} must be calculated from dc analysis

. DC Analysis



. KVL①: $10 = 5k I_{EQ} + V_{EB}$

$10 = 5k\ I_{EQ} + 0.7 \Rightarrow \boxed{I_{EQ} = 1.86\ mA}$

. $h_{ib} = \frac{25.69\ mV}{1.86\ mA} = \boxed{13.98\ \Omega = h_{ib}}$

1) $A_V = \frac{V_o}{V_i}$

$V_o = i_o . 4k$

$i_o = h_{fb} . i_e$

$i_e = \frac{V_i}{h_{ib}}$

$\Rightarrow V_o = 1 . \frac{V_i}{13.98} . 4k \Rightarrow \boxed{286 = \frac{V_o}{V_i}}$

2) $A_i = \frac{i_o}{i_i}$

$i_o = h_{fb} \cdot i_e$

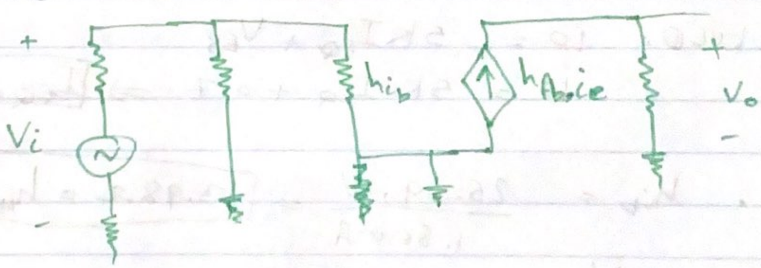
$i_o = i_e = \frac{5k}{5k + h_{ib}} \cdot i_i$

$\frac{i_o}{i_i} = \frac{5k\Omega}{5k\Omega + 13.98} < 1$

3) $Z_i = 5k \parallel h_{ib} = \frac{5k \cdot 13.98}{5k + 13.98} =$

4) $Z_o\Big|_{\text{ind. sources} = 0} = 4k\Omega$

b) with R_s



$i_i = \frac{V_i}{Z_i + R_s}$

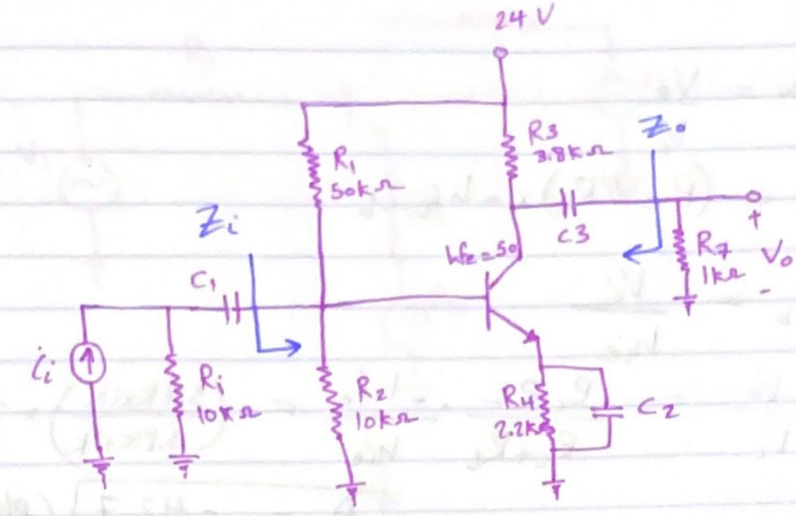
$i_e = \frac{5k}{5k + h_{ib}} \cdot i_i$

for $R_s = 50\Omega$
$A_V = 62.5$

for $R_s = 10k\Omega$
$A_V = 286$

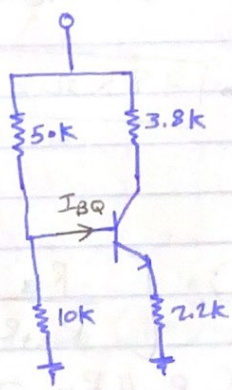
$\Rightarrow R_s \propto \frac{1}{A_V}$

ex



$$h_{ie} = \frac{V_T}{I_{BQ}}$$

To find I_{BQ}:



$$V_{TH} = \frac{10k}{10k + 50k} \cdot 24V$$

$$\boxed{V_{TH} = 4V}$$

$$R_{TH} = 10k // 50k$$

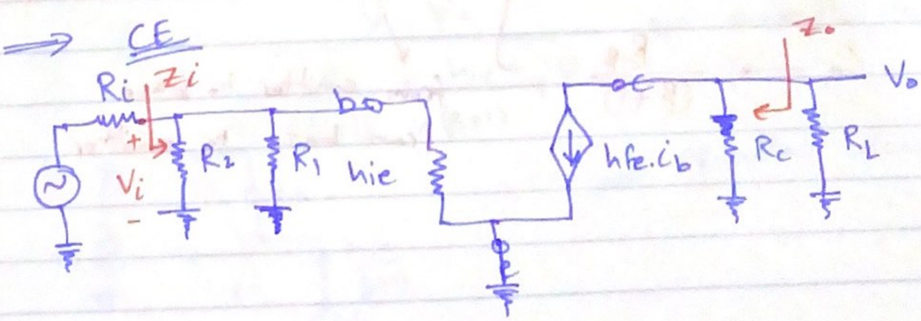
$$\boxed{R_{TH} = 8.33k\Omega}$$

$$I_E = I_B(\beta + 1)$$

$$I_B = \frac{V_{TH} - 0.7}{R_{TH} + R_E(\beta+1)} = \frac{4 - 0.7}{8.33k + 2.2k(50+1)}$$ (By KVL) Base equivalent circuit

$$I_E = \frac{V_{th} - 0.7}{R_E + \frac{R_{th}}{\beta+1}}$$ (emitter equivalent circuit)

→ CE



* $A_v = \frac{V_o}{V_i}$

$V_o = (R_c // R_L) \times -h_{fe} \cdot i_b$

$i_b = \frac{V_i}{h_{ie}}$

$\Rightarrow \frac{V_o}{V_i} = \frac{R_c R_L}{R_c + R_L} \cdot \frac{-h_{fe}}{h_{ie}} = \left(\frac{3.8k \times 1k}{3.8k + 1k}\right) \times \frac{-50}{928}$

$\boxed{A_V = -42.7}$ (phase shift 180°)

* $Z_i = (R_1 // R_2) // h_{ie}$

* $Z_o = R_3 = \boxed{3.8 \, k\Omega = Z_o}$

* $A_i = \frac{i_o}{i_i}$

$i_o = -h_{fe} \cdot i_b \left[\frac{R_3}{R_3 + R_7}\right]$

$i_b = i_i \left[\frac{R_1 // R_{th}}{(R_1 // R_{th}) + h_{ie}}\right]$

$A_i = \frac{i_o}{i_i} = \boxed{-33 = A_i}$

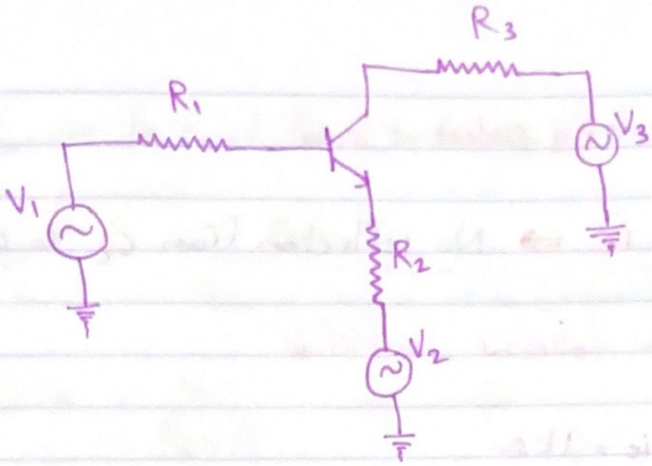
⇒ Impedence Reflection Concept

$i_b \longrightarrow R_E(\beta + 1)$ } Reflection → simplification technique

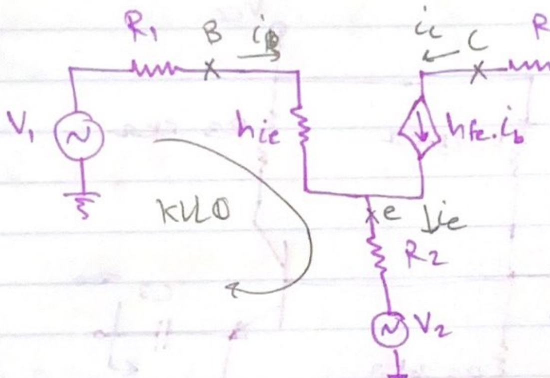
$i_e \longrightarrow \frac{R_B}{(\beta + 1)}$

from emitter to base

from base to emitter



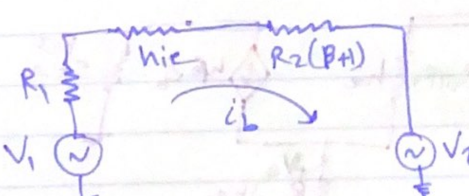
$\Downarrow$



$i_e = (\beta+1)\, i_b$

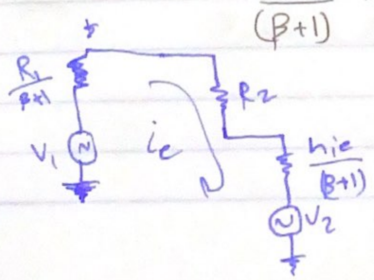
• $V_1 = R_1 i_b + hie.i_b + R_2 (\beta+1) i_b + V_2$

$$* \; i_b = \frac{V_1 - V_2}{R_1 + hie + R_2 \cdot (\beta+1)}$$ ⇐ base loop



equivalent circuit equation ⇐ reflection from Emitter to base

$$* \; i_e = \frac{V_1 - V_2}{\frac{R_1}{(\beta+1)} + \frac{hie}{(\beta+1)} + R_2}$$ ⇐ emmitter loop



⇐ equivelent circuit equation

→ Collector Equivalent Circuit

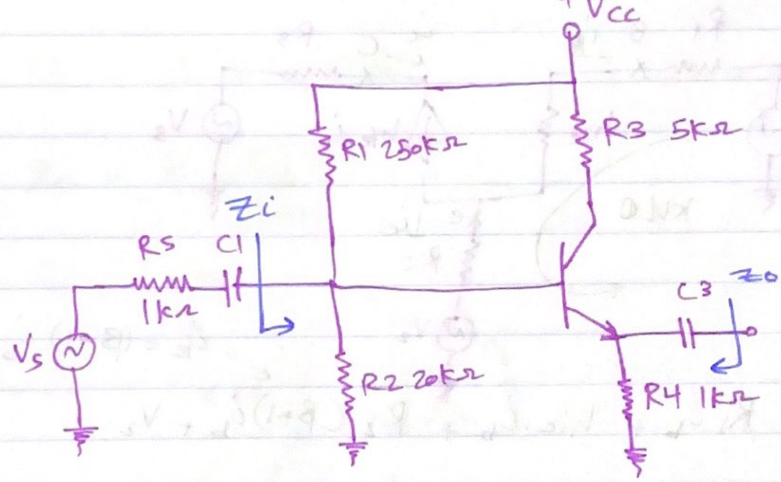
$i_c \simeq i_e \Rightarrow$ No reflection from i_c to i_e

→ Common Collector Amplifier

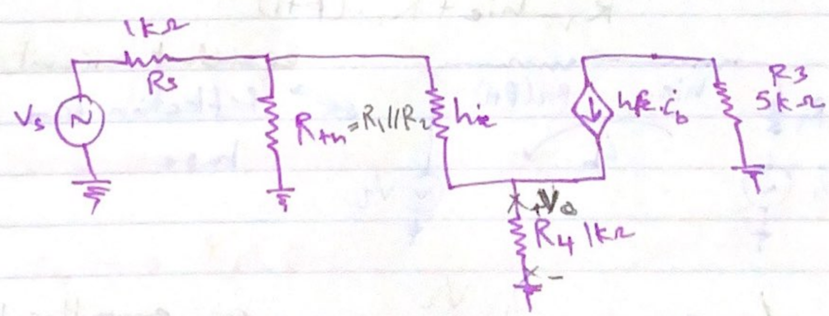
ex Given $h_{ie} = 1k\Omega$

$h_{fe} = \beta = 50$

Find A_v, A_i, Z_i, Z_o



⇓



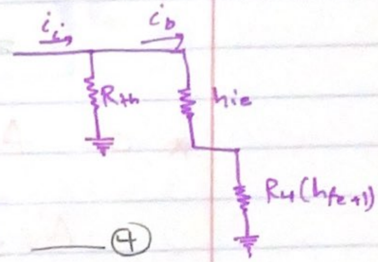
1) $V_o = 1k\Omega \cdot i_e$ —— ①

$i_e = i_b(h_{fe}+1)$

$i_b = ?$

i_b can be found from base equivalent circuit

$$i_b = i_i \cdot \frac{R_{th}}{R_{th} + h_{ie} + 1k\Omega(h_{fe}+1)} \quad \text{——— ②}$$



$$i_i = \frac{V_i}{R_s + Z_i} \quad \text{——— ③}$$

$$Z_i = R_{th} \,//\, (h_{ie} + 1k\Omega(h_{fe}+1)) \quad \text{——— ④}$$

$$A_V = \frac{V_o}{i_e} \times \frac{i_e}{i_b} \times \frac{i_b}{i_i} \times \frac{i_i}{V_s}$$

$$A_V = 0.915 < 1$$

* CC amplifier doesn't provide any voltage gain

$\Longrightarrow A_V \leq 1$

2) $A_i = \frac{i_o}{i_i}$

$$i_o = \frac{V_o}{1k\Omega}$$

$$i_o = i_e = i_b(h_{fe}+1)$$

$$i_b = i_i \frac{R_{th}}{R_{th} + (h_{ie} + 1k\Omega(h_{fe}+1))}$$

$\boxed{A_i = 13.39 > 1}$

Reflection from emitter to base

3) $Z_i = R_{th} \,//\, (h_{ie} + 1k\Omega(h_{fe}+1))$

$= 13.66\ k\Omega$ (high)

$$Z_o \Big|_{V_s=0} = \left[\left[\frac{R_s}{(h_{fe}+1)} \,//\, \frac{R_{th}}{(h_{fe}+1)}\right] + \frac{h_{fe}}{(h_{fe}+1)}\right] // 1k\Omega$$

reflection

مطلوب الي مسحبة ال emitter

$= 36.8\ \Omega$ (low)

* CC Amplifier

$A_v \leq 1$

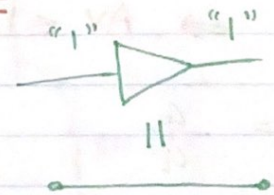
$A_i \gg 1$

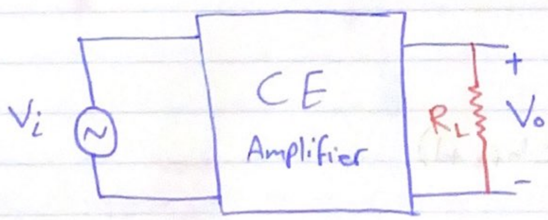
$Z_i \uparrow\uparrow$ $k\Omega$'s

$Z_o \downarrow\downarrow$ Ω's

⇒ CC amplifier as a buffer

. used to remove loading effect





without Load R_L

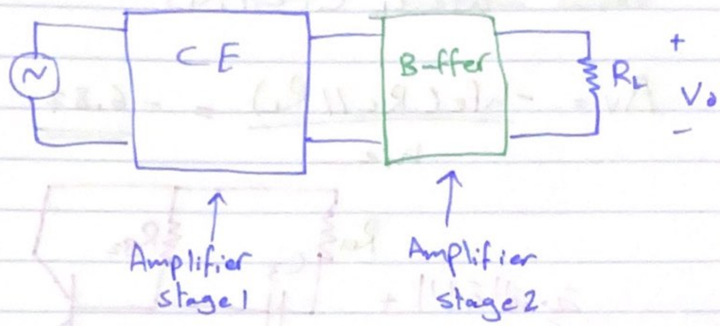
$$A_{VN} = \frac{V_o}{V_i}\Big|_{R_L=\infty}$$

$\neq$

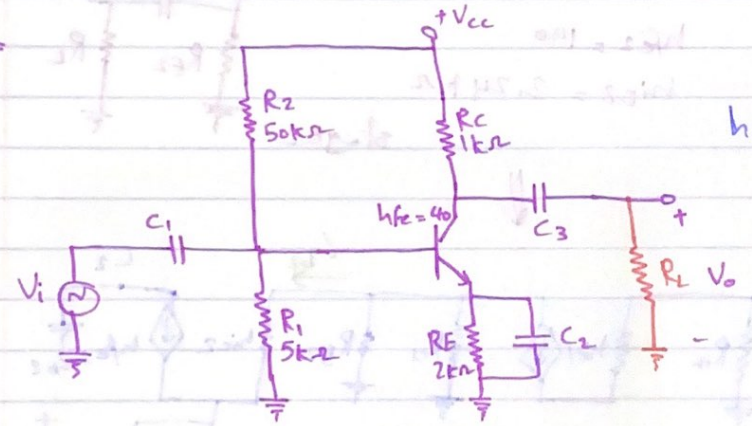
$$A_{VL} = \frac{V_o}{V_i}\Big|_{R_L\neq\infty}$$

$A_{VL} \ll A_{VN}$ (This is called loading effect)

- A solution is to add a buffer between the original Amplifier and the load

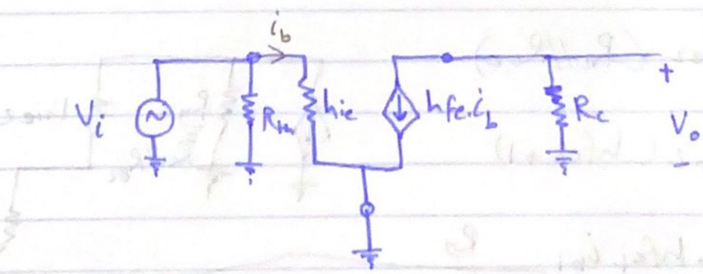


ex



hie = 1K

1) without $R_L \Rightarrow R_L = \infty$



$*\ V_o = -h_{fe} \cdot i_b \cdot R_c$

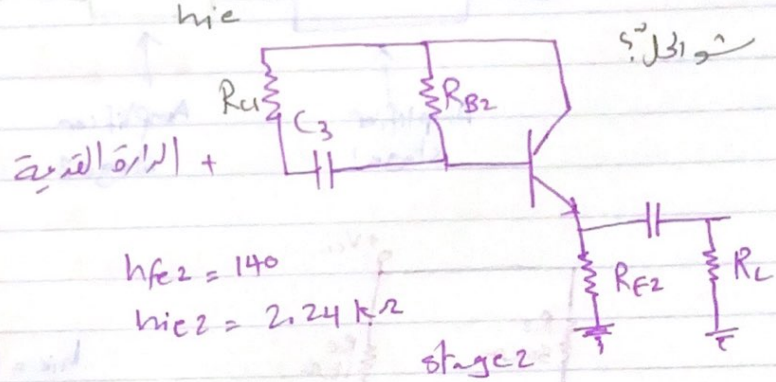
$$i_b = \frac{V_i}{h_{ie}}$$

$$A_v = \frac{V_o}{i_b} \cdot \frac{i_b}{V_i} = \frac{-h_{fe} \cdot R_c}{h_{ie}} = -140$$

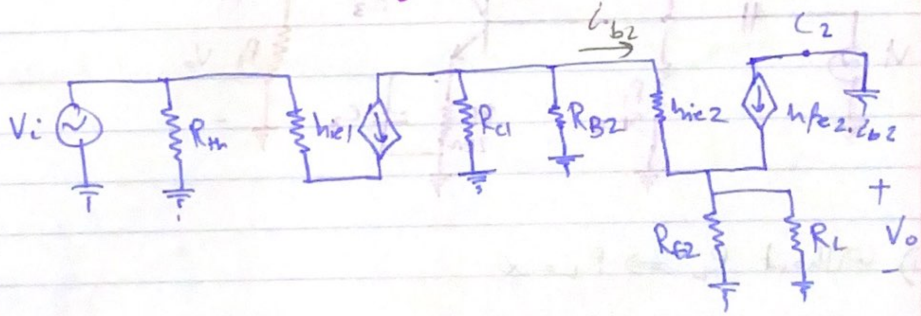
2) with $R_L = 50\,\Omega$

$$V_o = -h_{fe} \cdot i_b \,(R_C // R_L)$$

$$A_V = \frac{-h_{fe}(R_C//R_L)}{h_{ie}} = \underline{\underline{-6.87}}$$

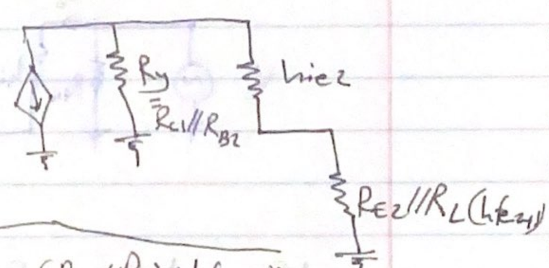


$\Downarrow$



$$V_o = i_{e2}\,(R_L // R_{E2})$$

$$i_{e2} = i_{b2}\,(h_{fe2}+1)$$



$$i_{b2} = -h_{fe1} \cdot i_{b1} \frac{R_y}{R_y + (h_{ie2} + (R_{E2}//R_L)(h_{fe2}+1))}$$

$$i_{b1} = \frac{V_i}{h_{ie1}}$$

$$A_V = \frac{V_o}{i_{e2}} \cdot \overset{①}{\frac{i_{e2}}{i_{b2}}} \cdot \overset{②}{\frac{i_{b2}}{i_{b1}}} \cdot \overset{④}{\frac{i_{b1}}{V_i}} = -95.6$$

CE + CC
multistage Amplifier ⟵ This is much better that without Buffer